The Eleventh Annual

# *Harriett J. Walton Symposium*

on

# Undergraduate Mathematics Research

## Program and Abstracts

Saturday, April 6, 2013

The Eleventh Annual

# *Harriett J. Walton Symposium*

on

# Undergraduate Mathematics Research

Sponsored by

The Mathematical Association of America (MAA) Regional Undergraduate Mathematics Conference Program through National Science Foundation Grant DMS-0846477

The Department of Mathematics
Morehouse College

The Division of Science and Mathematics
Morehouse College

Morehouse College

Saturday, April 6, 2013

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**John Silvanus Wilson Jr.**
President

April 6, 2013

Dear Symposium Participants:

On behalf of the College community, I welcome you to the Morehouse campus to participate in the Eleventh Annual Harriet J. Walton Symposium on Undergraduate Mathematics Research. Dr. Walton, a widely respected mathematics professor, encouraged students to excel in applied and research mathematics and continues to be a great champion of the discipline. Additionally, she provided for us an example of scholarly excellence and dedication to teaching mathematics.

It is our privilege to host the symposium, and we hope that your experience will prove intellectually provocative and inspiring. In the same manner that Dr. Walton approached learning, I challenge you to immerse yourself in this opportunity to exchange ideas and concepts with your colleagues.

I wish for each of you a thought-provoking meeting and an enjoyable visit.

Sincerely,

John Silvanus Wilson Jr.

MOREHOUSE COLLEGE

OFFICE OF THE PROVOST AND SENIOR VICE PRESIDENT FOR ACADEMIC AFFAIRS

Willis B. Sheftall, Jr.

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March 28, 2013

To Those Attending the Annual Harriet J. Walton Symposium:

It is my great pleasure to endorse the Eleventh Annual Harriet J. Walton Symposium on Undergraduate Mathematics Research. I am delighted that the Department of Mathematics has chosen to honor an esteemed colleague in such a manner. Dr. Walton brought skill and enthusiasm to her classroom and touched the lives of thousands of Morehouse men with her dedication and compassion. With its focus on mathematics research by undergraduates, this Symposium helps to ensure the continued intellectual growth of the students about whom Dr. Walton cares so much.

Best wishes for a rewarding experience.

Respectfully,

Willis B. Sheftall, Jr.

MOREHOUSE COLLEGE

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April 2, 2013

Dear Student Presenters and Colleagues:

On behalf of the Division of Science and Mathematics at Morehouse College, I want to welcome you to the Eleventh Annual Harriett Walton Symposium. To the students, I would encourage your continued participation in research, whether you intend to pursue a research career or not. It is one of the best means of developing your creative thinking and analytical reasoning skills, both of which are important goals of education. To the faculty, I want to express my thanks for your commitment to providing students with opportunities to engage in active learning through research. In the years ahead, research involving students and faculty will play an increasing role in American higher education. I hope that all of you will continue to be leaders in this important initiative.

For a very long time we, at Morehouse, have sought to enhance interest among our students in careers involving science and mathematics. In light of the fact that African-Americans continue to receive 2% or less of the Ph.D.s awarded in science, engineering and mathematics, it is evident that America and particularly HBCUs must redouble efforts to address this problem. We must continue to nurture and guide our students so that African-Americans can achieve parity in this very rewarding career, which is vital to the national interest. To this end, we established the Division of Science and Mathematics, an administrative unit that will help to remove the walls between departments, so that students in the sciences and mathematics will receive a first-rate, interdisciplinary education, which is consistent with the way that the most exciting scientific research is being done today.

I can think of no better person to honor than Harriett Walton, who over the course of 42 years was an excellent mathematics teacher and valued member of the Morehouse community. I am pleased to count myself among her former students.

I wish all of the students who are sharing the results of their research today, much continued success in your work, and hope that many of you will pursue a career in research.

Sincerely,

J.K. Haynes

J. K. Haynes, Ph.D.
David Packard Professor and
Dean of Science and Mathematics

re/JKH

MOREHOUSE
COLLEGE

DEPARTMENT OF MATHEMATICS

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6 April 2013

Dear symposium attendees and presenters:

We are happy to have you participate in this Eleventh Annual Harriett J. Walton Symposium on Undergraduate Mathematics Research. Since 2003, this symposium has become a highlight in the academic year for this region's mathematics students and a valuable opportunity for them to synthesize their research experiences, hone their presentation skills, and share the results of their work with each other and with the accompanying faculty and guests.

We in the Morehouse College Department of Mathematics are appreciative of the work you have done and of your travels, in many instances, to join us today. Undergraduate research like that presented at today's symposium serves to motivate and inform students about possibilities beyond the Bachelor's degree and to develop skills and habits of mind that can benefit them in graduate study and beyond.

Thank you for your participation this year, and we hope you will continue your support, joining us again in 2014 and beyond.

Duane Cooper
Assoc. Professor and Chair
Department of Mathematics

# Professor Harriett J. Walton

In September 1958, Harriett J. Walton joined the faculty of Morehouse College during the presidency of Benjamin Elijah Mays. She became a member of a team of three persons in the Department of Mathematics where she worked with the legendary Claude B. Dansby who served as Department Chair. Dr. Walton and her two colleagues taught all of the mathematics for the majors as well as the mathematics for non-science students. Dr. Walton relates that two of her favorite courses that she taught during this period were Abstract Algebra and Number Theory. The three-member mathematics department did an excellent job of preparing their mathematics majors for graduate school and the other students for success in their respective disciplines. In fact it was during this period of history that Morehouse gained the reputation of being an outstanding Institution especially for African American men. As the department grew, Dr. Walton shifted her attention away from mathematics majors and began to concentrate on students who needed special attention and care in order to succeed in mathematics. She became an advisor, mentor, tutor and nurturer to a large number of students matriculating at Morehouse College. Because of the caring attitude that she had for her students, some of them to this day refer to her as “Mother Walton.”

Dr. Walton has never been satisfied with mediocrity. Throughout her teaching career she demonstrated a love for learning. In 1958 when she arrived at Morehouse College she had an undergraduate degree in mathematics from Clark College in Atlanta, Georgia, a Master of Science degree in mathematics from Howard University, Washington D.C., and a second Master's degree in mathematics from Syracuse University. While at Morehouse teaching full time and raising a family of four children, Dr. Walton earned the Ph.D. degree in Mathematics Education from Georgia State University. After receiving her doctorate, Dr. Walton realized the emerging importance of the computer in education so she returned to school and in 1989 earned a Master’s degree in Computer Science from Atlanta University. She is indeed a remarkable person.

Dr. Walton’s list of professional activities, awards and accomplishments during her career is very impressive and too lengthy to be enumerated here. However a few special ones are her memberships in Alpha Kappa Mu, Beta Kappa Chi, Pi Mu Epsilon, and the prestigious Phi Beta Kappa Honor Society. Additionally she was selected as a Fulbright Fellow to visit Ghana and Cameroon in West Africa. Dr. Walton’s professional memberships included the American Mathematical Society, the Mathematical Association of America, National Council of Teachers of Mathematics (NCTM) and the National Association of Mathematicians (NAM). She served as Secretary/Treasurer of NAM for ten years. In May 2000, Dr. Walton retired from Morehouse College after forty-two years of service.

# Foreword

The Department of Mathematics and the Division of Science and Mathematics of Morehouse College would like to thank the student presenters and their advisors for their participation in the Eleventh Annual Harriett J. Walton Symposium on Undergraduate Mathematics Research. The Symposium is funded partially though the generous support of the Mathematical Association of America (MAA) Regional Undergraduate Mathematics Conference Program through National Science Foundation Grant DMS-0846477. The purposes of the Symposium are the following:

- to encourage students to do more undergraduate mathematics research
- to introduce students to their peers from various institutions and related fields
- to stimulate student interest in pursuing graduate degrees in mathematics and science
- to give students experience in presenting their research, both orally and in written form

To all supporters, thank you for your help to make the Eleventh Annual Harriett J. Walton Symposium on Undergraduate Mathematics Research a success. We hope to continue this event for many years to come.

# Symposium Committee

Abdelkrim Brania
Duane Cooper
Rudy L. Horne
Kiandra Johnson
Tuwaner Lamar
Benedict Nmah, Conference Director
Steve Pederson
Chuang Peng
Charles Phifer
Masilamani Sambandham
Ulrica Wilson
Chaohui Zhang

# Session Moderators

Rudy L. Horne
Kiandra Johnson
Tuwaner Lamar
Charles Phifer
Ulrica Wilson
George Yuhasz

# Proceedings Editors

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Rudy L. Horne
Benedict Nmah, Managing Editor

# Administrative Assistant

William Barnville

The Eleventh Annual

*Harriett J. Walton*

Symposium on Undergraduate Mathematics Research

Saturday, April 6, 2013

# Schedule

11:00 am - 11:20 am Welcome (Dansby Hall, Room #200)

11:25 am - 12:10 pm Student Presentations

12:15 pm - 1:15 pm Lunch

1:25 pm – 4:15 pm Student Presentations

4:20 pm - 4:30 pm Closing

## Session 1: Student Presenations

## 1. Dansby Hall, Room #300

11:25 am - 11:45 am, **Aaron Reaves,**
Morehouse College
*Analysis of a System of Nonlinear Boundary Value Problems*
*With Lidstone Boundary Conditions*

11:50 am - 12:10 pm, **Victor Bailey**,
Georgia State University
*4 x 4 Irreducible sign patterns that require four distinct eigenvalues*

## 2. Dansby Hall, Room #302

11:25 am - 11:45 am, **Samuel Dillow,**
Georgia State University
*The zero-forcing numbers of signed complete graphs and signed complete*
*Bipartite graphs*

11:50 am – 12:10 pm, **Curtis Clark Jr.,**
Morehouse College
*Integral Equations of Fredholm Type*

## 3. Dansby Hall, Room #306

11:25 am - 11:45 am, **Melissa O. Miller,**
Spelman College
*Queen Dido's Area Problem*

11:50 am – 12:10 pm, **Joseph Crawford**
Morehouse College
*Simple groups of order* $n \leq 100$

## Session 2: Student Presentations

## 1. Dansby Hall, Room #300

1:25 pm - 1:45 pm, **Jeremy Ariche,**
Morehouse College
*Methods for Finding Ramsey Numbers*

1:50 pm - 2:10 pm, **Jessica Gordon**,
Georgia State University
*The zero-forcing numbers of signed Moebius ladders*

2:15 pm - 2:35 pm, **Herns Mesamours**,
Albany State University
*Black-Scholes Model and Its Applications to Option Pricing and Derivatives*

## 2. Dansby Hall, Room #302

1:25 pm - 1:45 pm, **Kheri Hicks,**
Albany State University
*An Innovative Tax Collection Model For Revenue Maximization*

1:50 pm - 2:10 pm, **Joel Coppadge**,
Morehouse College
*Fractional Differential Equations*

2:15 pm - 2:35 pm, **Jordan Culliver, Courtney Mauck and Shawn Morris**,
Birmingham-Southern College
*A Mathematical Model for Solving Jigsaw Sudoku Puzzles*

## 3. Dansby Hall, Room #306

1:25 pm - 1:45 pm, **Willtresca Heppard and Herns Mersamours,**
Albany State University
*Solutions of Singularly Perturbed Third-Order Ordinary Differential Equations*

1:50 pm - 2:10 pm, **Zollie White III,**
Morehouse College
*Calculating Grobner Bases in a Polynomial Ring*

2:15 pm - 2:35 pm, **Laura Parrish**,
Clayton State University
*On Opinionated Complete Bipartite Graphs*

# Session 3: Student Presentations

## 1. Dansby Hall, Room #300

2:40 pm – 3:00 pm, **Zachary Richards, Aaron Sherrill and Jean Damascene Rugamba,**
Birmingham-Southern College
*Extending the Mathematical Study of Bracketology*

3:05 pm - 3:25 pm, **Justin C. McKenzie**,
Albany State University
*On Crytographic Algorithms and their implementation*

3:30 pm - 3:50 pm, **Anthony J. Baker and Zachary L. Williams**,
Paine College
*Asymptotic Connectivity of Multihop Wireless Networks with the Log-normal Shadowing Model*

## 2. Dansby Hall, Room #302

2:40 pm – 3:00 pm, **Michael Ngo, Petra President and ThuHong Nguyen**
Clayton State University
*Small Pattern Gallai Ramsey Numbers*

3:05 pm - 3:25 pm, **Michael Gray and Nicklaus Lynch**,
Birmingham-Southern College
*A Mathematical Study on a Modified Lights Out Cube*

3:30 pm - 3:50 pm, **Kheri Hicks, Erika Brown, Calhoun Marquis and Ketchup Nathan,**
Albany State University
*A Statistical Analysis of Network Penetration Testing in Information Assurance*

3:55 pm - 4:15 pm, **Michael J. English and Kevin R. Bowman**,
Morehouse College
*Product of Several Two-level Toeplitz Operators*

## 3. Dansby Hall, Room #306

2:40 pm – 3:00 pm, **John Zukley, Lauren Reibe and Taylor O'Daniel**
Birmingham-Southern College
*The Optimal Break: A Study of 2-Dimensional Pool*

3:05 pm - 3:25 pm, **Kelvin Williams**,
Albany State University
*Approximation of functions and their integrals using Newton's Interpolating Polynomials and associated binomial series*

3:30 pm - 3:50 pm, **Pritul Patel**,
Clayton State University
*On the Edge-Balanced Index Set of Complete Odd Bipartite Graphs*

3:55 pm - 4:15 pm, **Laporchia Grier and Audrianna Rucker**,
Albany State University
*Power Series Expansion Procedure and Lessons from the History of Mathematics*

# Abstracts

**Jeremy Ariche,** Department of Mathematics, Morehouse College
Title: Methods for Finding Ramsey Numbers
Advisor: Dr. Tuwaner Lamar

A Ramsey number R(s,t), where both s and t are positive integers, is defined as the lowest possible number of n vertices in a graph G where it contains a mutually adjacent sub-graph denoted as $K_s$ and a mutually non-adjacent sub-graph denoted as $K_t$. It is understood that a Ramsey number can be bounded between two numbers. The process of finding a Ramsey number is difficult. After gaining a complete understanding of what Ramsey numbers are, one must begin to investigate methods of how previous Ramsey numbers were discovered. The proofs of them are rigorous. Once one understands how previous Ramsey numbers were discovered, one can begin to utilize similar methods. However, as s and t become larger, the methods become more difficult to utilize. The methods in this paper involve narrowing the upper and lower bounds of the Ramsey number until they are equal. The Ramsey number of interest is R(5,5).

**Victor Bailey,** Department of Mathematics, Georgia State University
Title: 4 x 4 Irreducible sign patterns that require four distinct eigenvalues
Advisor: Dr. Zhongshan Li

A sign pattern matrix is a matrix whose entries are from the Set $\{+, -, 0\}$. An $n \times n$ sign pattern matrix A requires all distinct eigenvalues if every real matrix whose entries have signs given by the corresponding entries of A, sign pattern represented by sgn(A), has n distinct eigenvalues. For n = 2 and n = 3, all sign patterns requiring all distinct eigenvalues have been determined. However, the case of n = 4 is still not completely solved. We determine more necessary and/or sufficient conditions for $4 \times 4$ sign patterns to require all distinct eigenvalues.

**Anthony J. Baker and Zachary L. Williams,** Department of Mathematics, Science and Technology, Paine College
Title: Asymptotic Connectivity of Multihop Wireless Networks with the Log-normal Shadowing Model
Advisor: Dr. Lixin Wang

The unit-disk communication model of wireless networks is based on the path loss phenomenon alone and assumes that the received signal strength at a receiving node from a transmitting node is only determined by a deterministic function of the Euclidean distance between the two nodes. With randomly-deployed wireless networks under such a simple communication model, the vanishment of isolated nodes asymptotically implies connectivity of networks. However, in reality, the received signal strength often shows probabilistic variations induced by shadowing effects that are unavoidably caused by different levels of clutter (e.g., various background noises and obstructions) on the propagation path. To better capture physical reality, one should consider the variations of

the received signal strength. It has been shown that a more accurate modeling of the physical layer is indeed important for better understanding of multihop wireless network characteristics. This generalized radio propagation model is referred to as the *log-normal shadowing model* which has been widely used by many researchers.
In this paper we study the connectivity of multihop wireless networks with such a realistic model by giving necessary and sufficient conditions for the asymptotic vanishing of the isolated nodes in the wireless network under the log-normal shadowing model. The vanishing of isolated nodes is not only a prerequisite but also a good indication of network connectivity. All previous known work on network connectivity under such a model were obtained only based on simulation studies or ignoring the important boundary effect to avoid the challenging technical analysis, and thus cannot be applied to any practical wireless networks. It is extremely challenging to take the complicated boundary effect into consideration under such a realistic model because the transmission area of each node is an irregular region other than a circular area. To the best of our knowledge, there are no theoretical results obtained by rigorous analytical studies for connectivity of multihop wireless networks under such a realistic model when the boundary effect is taken into consideration. We assume the wireless nodes are represented by a Poisson point process with density $n$ over a unit-area disk, and two nodes are directly connected if the received power at a receiving node under such a realistic model is not less than some given threshold. Under certain mild assumption, we derived necessary and sufficient conditions for the asymptotic vanishing of the isolated nodes in the wireless network under the log-normal shadowing model with the complicated boundary effect taken into consideration as the density $n$ goes to infinity, and thus certain level of network connectivity can be expected when $n$ is sufficiently large. The Palm theory for Poisson processes is utilized to derive the necessary and sufficient conditions for the asymptotic vanishing of the isolated nodes.

**Curtis Clark Jr.,** Department of Mathematics, Morehouse College
Title: Integral Equations of Fredholm Type
Advisor: Dr. A. Brania

In this study, we investigate the solution of linear integral equations of Fredholm Type with finite rank kernels. We use finite kernels to approximate integral equations of Fredholm Type with other types of kernels. This method will be ultimately used as I further study integral equations with Fuzzy Numbers.

**Joel Coppadge,** Department of Mathematics, Morehouse College
Title: Fractional Differential Equations
Advisor: Dr. Masilamani Sambandham

Fractional differential equations are derivatives or integrals of fractional order instead of integer order. Using Caputo's fractional differential equation, we develop numerical methods. In particular for Caputo's fractional differential equation, we will develop the Improved Euler method for fractional derivatives.
**Joseph Crawford,** Department of Mathematics, Morehouse College
Title: Simple groups of order $n \leq 100$

Advisor: Dr. Ulrica Wilson

In this project we will find all of the simple groups of order $n \leq 100$. We note the abelian case will be covered quickly in the introduction and that the problem becomes much more interesting and difficult in the nonabelian case. For this talk, our strategy lies primarily in proving that every nonabelian group of order $n \leq 100$ is not simple except when $n \neq 60$.

**Jordan Culliver, Courtney Mauck and Shawn Morris,** Department of Mathematics,
Birmingham-Southern College
Title: A Mathematical Model for Solving Jigsaw Sudoku Puzzles
Advisor: Dr. Bernadette Mullins

The Sudoku puzzle is the composite result of the study of magic squares, Latin squares and gerechte designes over multiple centuries. We provide a brief history of this study, as well as an outline of the rules of different variations of the puzzle, including both traditional and Jigsaw Sudoku. We explain how Arnold, Lucas and Taalman showed that the Grobner basis can be used to characterize the solution of a traditional Sudoku puzzle. Bartlett and Langville created an integer programming model that provides a solution to a traditional Sudoku puzzle by mathematically implementing the rules of the puzzle. By modifying this model to conform to the rules of Jigsaw Sudoku, we are able to produce solutions to Jigsaw Sudoku puzzles.

**Samuel Dillow,** Department of Mathematics,
Georgia State University
Title: The zero-forcing numbers of signed complete graphs and signed complete bipartite graphs
Advisor: Dr. Hendricus Van der Holst

A signed graph is a graph where each edge has been assigned + or −. Suppose we colored black a subset S of the vertex set of the signed graph and colored white the other vertices of the signed graph. We are now allowed to color the white vertices black according to following the rules: if a subset of k vertices of S is adjacent to a set W of exactly k white vertices, and the k by k $\{+,-,0\}$-matrix defined by these two sets of k vertices has nonzero determinant, then we may color the vertices W black. These rules may now be applied to our new set of black vertices. If after repeatedly applying these rules we can end with all vertices black, then we call S a zero-forcing set. The zero-forcing number of a signed graph is the minimum cardinality of any zero-forcing set. A triangle in a complete graph is a cycle with 3 edges. A triangle is positive if it has 2 or 0 negative edges, otherwise it is negative.

We have shown that a signing of a complete graph on n vertices has zero-forcing number $n - 2$, unless in the complete graph each triangle is positive or each triangle is negative, in which cases the zero-forcing numbers are $n - 1$. We have also some results on the zero-forcing number of signings of complete bipartite graphs.

**Michael J. English and Kevin R. Bowman,** Department of Mathematics,
Morehouse College
**Title:** Product of Several Two-level Toeplitz Operators
Advisor: Dr. Selcuk Koyuncu

In this paper, we give necessary and sufficient conditions for when the product of two-level Toeplitz operators is again a Toeplitz operator. We then show that $T_f T_g = 0$ if and only if f or g is identically zero where $T_f$ and $T_g$ are two-level Toeplitz operators. Finally, we conjectured zero product of three two-level Toeplitz operators.

**Jessica Gordon,** Department of Mathematics,
Georgia State University
**Title:** The zero-forcing numbers of signed Moebius Ladders
Advisor: Dr. Marina Arav

Abstract: A signed graph is a graph where each edge has been assigned + or −. Suppose we colored black a subset S of the vertex set of the signed graph and colored white the other vertices of the signed graph. We are now allowed to color the white vertices black according to following the rules: if a subset of k vertices of S is adjacent to a set W of exactly k white vertices, and the k × k {+, −, 0}-matrix defined by these two sets of k vertices has nonzero determinant, then we may color the vertices W black. These rules may now be applied to our new set of black vertices. If after repeatedly applying these rules we an end with all vertices black, then we call S a zero-forcing set. The zero-forcing number of a signed graph is the minimum cardinality of any zero-forcing set. A cycle in a signed graph is positive if it has an even number of negative edges.

We have shown that a signing of the Moebius ladder on 6 vertices has zero-forcing number 3, unless each cycle is positive, in which case the zero-forcing number is 4. The zero-forcing number of the Moebius ladders on 2n vertices, where n > 4, is 4.

**Michael Gray and Nicklaus Lynch,** Department of Mathematics,
Birmingham-Southern College
**Title:** A Mathematical Study on a Modified Lights Out Cube
Advisor: Dr. Bernadette Mullins

The Lights out game has been around since 1981 and mathematicians have talked about and analyzed it in almost every way. We looked outside the box by completely redesigning the game thereby making a completely new one. By rewriting the rules matrix as established by Todd Feil and Marlow Anderson in Turing Lights Out with Linear Algebra, we have found that one can create all types of new patterns in order to enhance the game. If you take sides off and move them, then more degrees of freedom open up for the game and allow more solvable sets and challenging patterns to arise. Our modified cube consists of taking the top face off of the cube and placed it inside the now open box along the diagonal of the cube. Following the rules of the original Lights Out game, our modified cube gave us new winning patterns and solutions.

**Laporchia Grier and Audrianna Rucker,** Department of Mathematics and Computer Science, Albany State University
Title: Power Series Expansion Procedure and Lessons from the History of Mathematics
Advisor: Dr. Zephyrinus Okonkwo

**Abstract:**
Mathematical concepts developed in the ancient civilizations of Egypt, Greece, Babylon, India, and China provided exciting foundations for Modern Mathematics of today. In this study, we examines some of these methods developed in the past such as the Egyptian Doubling Method, determination of Pi, determination of the Area of a Circle, Square Roots of Integers, the Relationship between a Triangle and a Circle, and foundations of long multiplication. We will also study the power series expansion methods, the convergence of such series, and the associated history. Furthermore, we discuss how the historical perspective of these concepts can be integrated in lessons to bring excitement and enhance learning. Several examples will be presented for illustration.

**Willtresca Heppard and Hern Mersamours,** Department of Mathematics and Computer Science, Albany State University
Title: Solutions of Singularly Perturbed Third-Order Ordinary Differential Equations
Advisor: Dr. Zephyrinus Okonkwo

Singular perturbation problems arise in many fields of science and engineering including mechanical and electrical systems, fluid flow through pipes, and biological systems. In this paper, we present solutions to the singularly perturbed third order linear ordinary differential equation

$$(1) \quad \varepsilon\frac{d^3y}{dx^3}+a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=F(x),$$

where $a=a(x)$, $b=b(x)$, $c=c(x)$, and $\varepsilon>0$ is a small parameter.
Furthermore, we derive a closed-form solution to equation (1) and show that as $\varepsilon \to 0$, the solution of the third order singularly perturbed differential equation collapses to the solution of an associated second order differential equation. Examples are presented for illustration.

**Kheri Hicks,** Department of Mathematics and Computer Science, Albany State University
Title: An Innovative Tax Collection Model for Revenue Maximization
Advisor: Dr. Robert Steven Owor

Abstract: Even in most advanced countries, revenue collection is a challenge due to variation of prices of goods and services. In this paper, a computer model is developed for revenue collection from a variety of consumer items and commodity items. Consider a county which has m taxes and fees collection centers and n types of items on which taxes are levied, then the revenue generated is given by $\Gamma$(m, n).

Suppose $m = (m_1, m_2, \ldots, m_p)$ is a p-vector and $n = (n_1, n_2, \ldots, n_q)$ be a q-vector, then one can generate a $(p \times q)$ revenue matrix, denoted by $\Gamma(m, n)$ where each entry is of the form $f(m_i, n_j)$ for $i = 1, 2, \ldots, p$ and $j = 1,2, \ldots, q$.

Essentially, the total revenue can be written in the form :

$$(1.1) \quad \Gamma(m,n) = \sum_{j}^{q} \sum_{i=1}^{p} f(m_i, n_j)$$

Let us recall that the amount of revenue at any time depends on both time and probability. That is,

$$(1.2) \quad f = f(m_1, n_1, t, w).$$

We assume here that w is a Gaussian process. Hence equation (1.1) can be written in the form:

$$(1.3) \quad \Gamma(m, n) = \Sigma\, \Sigma\, f(m_i, n_j, t, w).$$

Consequently, the expected value of the revenue is given by

$$(1.4)\ E(\Gamma(m, n)) = E(\ \Sigma\, \Sigma\, f(m_i, n_j, t, w)\ ) .$$

Since $f(m_i, n_j, t, w)$ is a random variable, then it has a probability distribution. In this paper, an algorithm will be developed and a computer program written to implement an innovative tax collection model which will reduce inefficiencies and increase revenue accruing to the state.

**Kheri Hicks, Erika Brown, Calhoun Marquis and Ketchup Nathan,** Department of Mathematics and Computer Science, Albany State University
Title: A Statistical Analysis of Network Penetration Testing in Information Assurance
Advisor: Dr. Robert Steven Owor

***Abstract:*** A network penetration test, colloquially called (PenTest), is a method of investigating the vulnerabilities of a computer system or network by simulating several attacks from malicious external and internal attackers. Potential vulnerabilities are investigated, detected and exploited to gain entry into the system. These vulnerabilities may be due to poor or improper system configuration, both known and unknown hardware or software flaws, and operational weaknesses in procedural or technical specifications and implementations. The analysis is carried out from a potential attacker's point of view. Security flaws discovered during the process are presented to the organization in a report. Effective penetration testing combines this report with an accurate assessment of the potential impact to the organization, of different levels of attack.  A comprehensive range of technical and procedural countermeasures to reduce risks is also recommended by the report. In this paper we propose a statistical methodology to identify and analyze a set of computer systems' attack vectors by identifying higher-risk vulnerabilities that result from a combination of lower-risk vulnerabilities exploited in a possible set of sequences, discuss hidden vulnerabilities;

estimate the magnitude of potential business and operational impacts of successful attacks, and assess the ability of network defenders to successfully detect and respond to the attacks. Finally, the paper suggests further areas of work in this new, exciting and challenging field.

**Justin C. McKenzie,** Department of Mathematics and Computer Science,
Albany State University
Title: On Cryptographic Algorithms and Their Implementation
Advisor: Dr. Seyed Roosta

Abstract: The purpose of this project is to give insight into the field of study that allows people to send and receive information with confidence in its security and integrity. That field of study is cryptography. Cryptography is the science and practice of encrypting data into an unreadable format then decrypting it back to the original data. This research project in particular defines what cryptography is and briefly reports on its origins. It also describes the two systems (secret key and public key) that make up cryptography as a whole then explores the public key system and three of its algorithms, including the most popular one implemented as of today (RSA). RSA stands for Ron Rivest, Adi Shamir and Leonard Adleman who described algorithm in 1977. After an understanding of the system and its algorithms has been obtained, the RSA algorithm will be studied in detail and implemented. At the completion of the project we hope to have learned what makes a successful cryptography system and how the algorithms work. The results of this research will be used to educate the community on a system they use every day without knowing and present methods of ensuring their information remains secure.

**Herns Mesamours,** Department of Mathematics and Computer Science,
Albany State University
Title: Black-Scholes Model and Its Applications to Option Pricing and Derivatives
Advisor: Dr. Zephyrinus C. Okonkwo

**Abstract:** Mathematical financial models have become very popular since Dr. Fischer Black and Dr. Myron Scholes popularized the *Black-Scholes Model for Option Pricing* in 1973. Mathematical foundations needed to deeply understand and appreciate this model include analysis, probability theory, stochastic calculus, and stochastic differential equations. In this research, we derive the Black-Shores equations for option pricing using Taylor Series in two variables. Furthermore, we present elementary examples of options, which exemplify the dynamics of the securities market, futures, and other forms of derivatives.

**Melissa O. Miller.,** Department of Mathematics,
Spelman College
Title: Queen Dido's Area Problem
Advisor: Dr. Jeffrey Ehme

Queen Dido's was challenged with the problem of maximizing area while keeping the perimeter constant. Problems of this sort are now called isoperimetric problems. We will

mathematically model this problem and find a complete solution using the Calculus of Variations. The Calculus of Variations is a generalization of the ordinary calculus involving numbers to a calculus involving functions and curves.

**Michael Ngo,** Department of Mathematics, Clayton State University
Title: Small Pattern Gallai Ramsey Numbers
Advisor: Dr. Elliot Krop

The minimum order of any complete graph so that for any coloring of the edges by k colors it is impossible to avoid a monochromatic or rainbow triangles is known as the smallest Gallai-Ramsey number. For any graph H with edges colored from the above set of k colors, if we consider the condition of excluding H in the above definition, we produce a pattern Gallai-Ramsey number. In this talk, we consider this problem when H is a two-colored cycle with two consecutive colors and discuss the solution in terms of k.

**Pritul Patel,** Department of Mathematics, Clayton State University
Title: On the Edge-Balanced Index Sets of Complete Odd Bipartite Graphs
Advisor: Dr. Christopher Raridan

Abstract: We determine the edge-balanced index sets for all complete bipartite graphs with parts of odd order.

**Laura Parrish,** Department of Mathematics, Clayton State University
Title: On Opinionated Complete Bipartite Graphs
Advisor: Drs. Christian Barrientos and Elliot Krop

Let G be a graph with vertex set V(G) and edge set E(G), and f be a 0-1 labeling of E(G) so that the absolute difference in the number of edges labeled 1 and 0 is no more than one. Call such a labeling f edge-friendly. We say an edge-friendly labeling induces a partial vertex labeling if vertices which are incident to more edges labeled 1 than 0, are labeled 1, and vertices which are incident to more edges labeled 0 than 1, are labeled 0. Vertices that are incident to an equal number of edges of both labels we call unlabeled.
We call an edge-friendly labeling of a graph opinionated, if given the above definition, every vertex receives a label. We show that for every positive integers n, m and $K_{nm}$ is opinionated.

**Aaron Reaves,** Department of Mathematics, Morehouse College
Title: Analysis of a System of Nonlinear Boundary Value Problems with Lidstone Boundary Conditions
Advisor: Dr. Tuwaner Lamar

This project discusses the existence of positive solutions for a system of bending elastic Beam equations with Lidstone boundary conditions by using fixed point theory. In mechanics, the problem describes the deformation of elastic beams in equilibrium state, whose two ends are simply supported. This paper will establish the existence of solutions for three beams and then attempt to generalize the results for some n number of beams.

**Zachary Richards, Aaron Sherrill and Jean Damascene Rugamba,** Department of Mathematics, Birmingham-Southern College
Title: Extending the Mathematical Study of Bracketology
Advisor: Dr. Bernadette Mullins

Braketology is the process of predicting the results of the NCAA Division I men's basketball tournament and filling out a bracket based on the predictions. There are different mathematical methods to take on this prediction. A heavily studied formula for bracketology is the Colley Method. Our study, motivated by Tim Chartier's paper, "Bracketology: How Can Math Help", focuses on extending the Colley Method to construct a more accurate predictive tool for the tournament. We carried out this extension by incorporating three factors: away winning percentage minus home losing percentage, strength of schedule, and a time-based game weight. While adjusting the three factors, our extended Colley method based formula was used to calculate predictive brackets for the 2009 tournament. Out of our various methods tested, the weekly time-based weight paired with a scaling of our remaining two factors gave us our best predictive bracket for the tournament.

**Zollie White III,** Department of Mathematics, Morehouse College
Title: Calculating Grobner Bases in a Polynomial Ring
Advisor: Dr. Benedict Nmah

The concept of Grobner basis is relatively new. Though Bruno Buchberger first introduced the concept of Grobner basis in his doctoral thesis in 1965, the concept remained unknown to the research community until a decade later. The theory of Grobner basis has become an important subarea in computer algebra. It is included in all the major program systems of symbolic computation, and it is being fruitfully applied to a variety of research areas. Having developed a basic understanding of what a Grobner basis is, I calculated Grobner bases over given polynomial rings using the Multivariate Division Algorithm.

**Kelvin Williams,** Department of Mathematics and Computer Science, Albany State University
Title: Approximation of Functions and their Integrals using Newton's Interpolating Polynomials and associated Binomial Series
Advisor: Dr. Zephyrinus C. Okonkwo

**Abstract:** It is well-known that obtaining integrals of the form

$$\Psi = \int_a^b f(x)dx = \int_0^\pi \frac{2^x \sin x}{\sqrt{1-k^2 \sin^2 x}} dx, \tag{1}$$

in closed-form sometimes can be challenging. In this paper, we derive the Newton's Interpolating Polynomial approximation, $N_n(x)$ for the function

(2) $$\Im(x)=\frac{2^x \sin x}{\sqrt{1-k^2\sin^2 x}}$$

on the interval $[0,\pi]$. We show that that for every $x\in[0,\pi]$, $|\Im(x)-N_n(x)|<\varepsilon$. Furthermore, we use series expansion procedure to derive a Maclaurin Polynomial approximation, $M_n(x)$ for $\Im(x)$. Finally, we compare and discuss the accuracy of the integrals obtained by integrating $N_n(x)$ and $M_n(x)$ over the interval $[0,\pi]$.

**John Zukley, Lauren Reibe and Taylor O'Daniel,** Department of Mathematics,
Birmingham-Southern College
Title: The Optimal Break: A Study of 2-Dimensional Pool
Advisor: Dr. Bernadette Mullins

Abstract: The Optimal Break in 2-Dimensional Pool studies the fundamentals of collisions between billiard balls through mathematical application. Beginning with a definition for an optimal break and an exploration of physical principles, we are able to arrive at equations for the velocities of each ball in the 8-ball formation under the assumption that there is no friction and we are in 2-dimensions. Using these equations in Excel, we are able to determine which balls have the potential to meet pockets when the head ball transfers momentum from various angles. We find an angle at which to hit the rack so the 8-ball will meet a pocket (winning the game) and begin exploring the physical properties of 3-dimensional pool, friction, and their effects on the break.